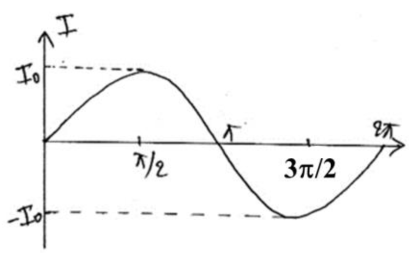
Alternating Current

Alternating current

A type of current or voltage whose magnitude changes with time and direction reverse periodically is known as alternating current or alternating voltage.

The instantaneous value of alternating current is

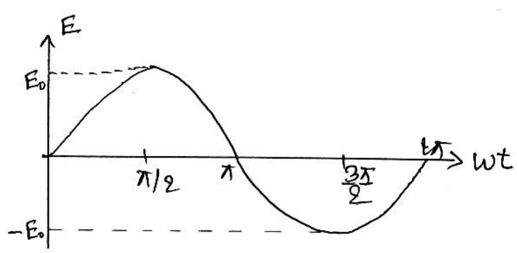
$I = I_o \sin \omega t.$



Where I_o is the maximum (peak) value of current.

The instantaneous value of alternating voltage is

$E = E_o \sin \omega t.$



Where E_o is maximum (peak) value of voltage (emf)

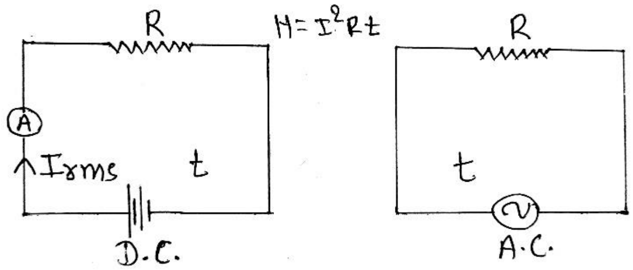
Measurement of alternating current

1. The average value of alternating current measured in complete cycle is zero.
2. The average value of alternating current measured in half cycle is known as average value. The average value of alternating current is

   $I_{Avg.} = 0.637\ I_o$

   This value is not in good agreement with practical value.

3. Root mean square value of current ($I_{r.m.s}$)



Root Mean Square value of current is defined as the steady current (D.C.) which develops heat in a given resistance (R) for a given time (t) as that done by alternating current for the same resistance in same time. It is denoted by I_{rms}.

Let us consider a resistor of resistance R is connected with an AC source. Let $I = I_o \sin \omega t$ be the instantaneous value of current flowing through the resistance. The small amount of work done by the alternating current in small time dt is

$$dw = I^2 Rdt$$

or, $$dw = I_o^2 \sin^2 \omega t \, Rdt$$

or, $$dw = I_o^2 R \sin^2 \omega t \, dt \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{(i)}$$

The total amount of work done in a complete cycle can be obtained by integrating equation (i) from limit O to T.

$$W = \int_O^T I_o^2 R \sin^2 \omega t \, dt$$

or, $$W = I_o^2 R \int_O^T \sin^2 \omega t \, dt$$

or, $$W = I_o^2 R \int_O^T \left(\frac{1 - \cos 2\omega t}{2}\right) dt$$

or, $$W = \frac{I_o^2 R}{2} \left[\int_O^T dt - \int_O^T \cos 2\omega t \, dt\right]$$

or, $$W = \frac{I_o^2 R}{2} [T - O] \quad \left(\because \int_O^T \cos 2\omega t \, dt = 0\right)$$

or, $$W = \frac{I_o^2 RT}{2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(ii)}$$

Let I_{rms} be the steady value of current which is passed through the same-resistance 'R' for same time 't'. Now heat developed by the current is

$$H = I_{rms}^2 \, RT \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(iii)}$$

According to the definition,

$$H = W$$

or, $$I_{rms}^2 RT = \frac{I_o^2 RT}{2}$$

or, $$I_{rms}^2 = \frac{I_o^2}{2}$$

or, $I_{rms} = \frac{I_o}{\sqrt{2}}$

$\therefore$ $I_{rms} = 0.7070\ I_o$

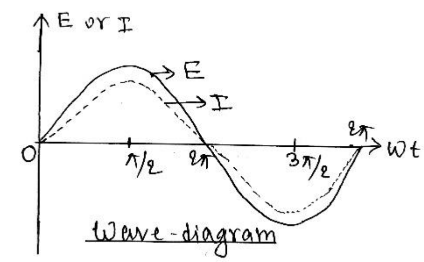
Similarly,

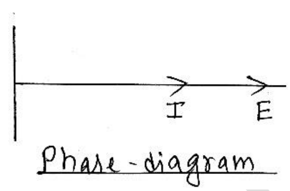
$E_{rms} = 0.707\ E_o$

This value is in good agreement with practical value.

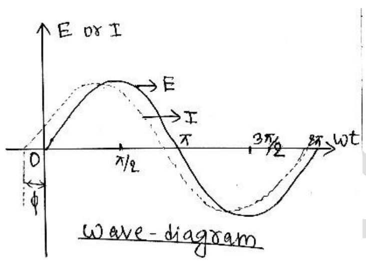
Wave diagram and phase diagram.

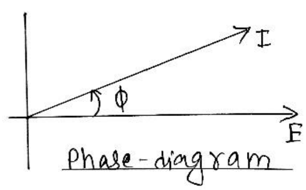
(i) $E = E_o \sin \omega t$ and $I = I_o \sin \omega t$



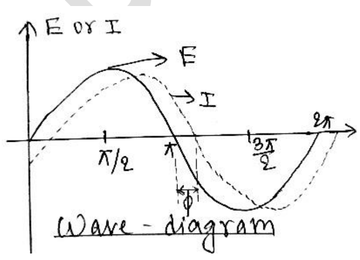


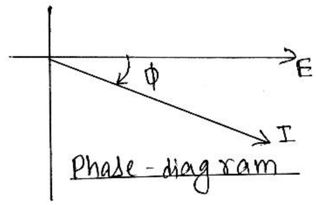
(ii) $E = E_o \sin \omega t$ and $I = I_o \sin (\omega t + \phi)$





(iii) $E = E_o \sin \omega t$ and $I = I_o \sin (\omega t - \phi)$





AC circuit containing resistor only.

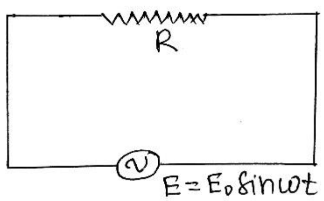


Fig: AC circuit containing resistor only.

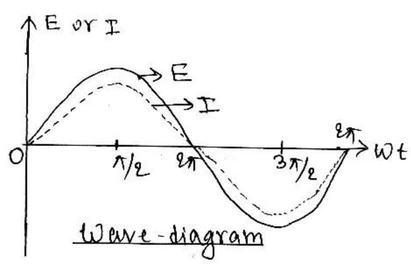
Let us consider a resistor of resistance R is connected with an AC source of emf $E = E_o \sin \omega t$ where E is instantaneous value of emf and E_o is peak value of emf. The instantaneous value of current flowing through the circuit is

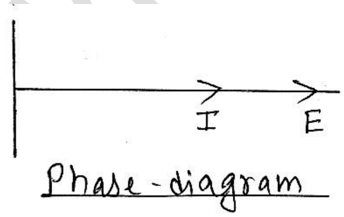
$$I = \frac{E}{R}$$

Or, $$I = \frac{E_o \sin \omega t}{R}$$

Or, $$I = I_o \text{ sini } \omega t \qquad \left(\because \quad I_o = \frac{E_o}{R}\right)$$

Since we are started from $E = E_o \sin \omega t$ and arrived at $I = I_o \sin \omega t$, the wave and phase diagram are:





AC circuit containing inductor only.

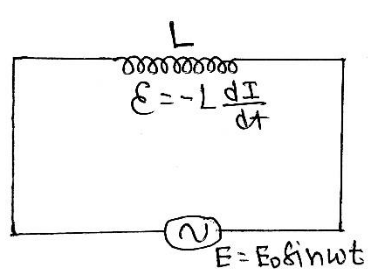


Fig: AC circuit containing inductor only.

Let us consider an inductor of inductance L is connected with an AC source of emf $E = E_o \sin \omega t$. When the inductor is connected with AC source, changing current flows through it due to which an amount of emf induced in the coil. Let the emf induced in the coil is

$$\varepsilon = -L\frac{dI}{dt}$$

From Kirchhoff's second law (KVL),

$$E + \varepsilon = O$$

Or, $$E - L\frac{dI}{dt} = 0$$

Or, $$E_o \sin\omega t = L\frac{dI}{dt}$$

Or, $$dI = \frac{E_o}{L}\sin \omega t \, dt.$$

On integrating above equation; we get;

$$\int dI = \frac{E_o}{L}\int \sin \omega t \, dt$$

Or, $$I = \frac{E_o}{L}\left(-\frac{\cos \omega t}{\omega}\right)$$

Or, $$I = -\frac{E_o}{\omega L}\cos \omega t$$

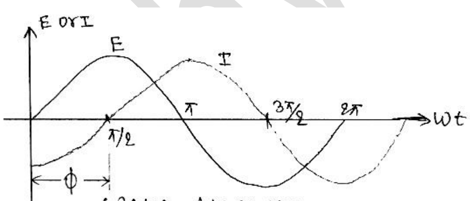
Or, $$I = -\frac{E_o}{X_L}\cos \omega t$$ (Where $X_L = \omega L$ is inductive reactance i.e. total resistance offered by inductor.)

Or, $$I = -I_o \cos \omega t$$

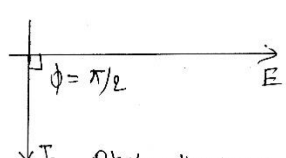
Or, $$I = -I_o \sin\left(\frac{\pi}{2} - \omega t\right)$$

Or, $$I = I_o \sin\left(\omega t - \frac{\pi}{2}\right)$$

Since we have started from $E = E_o \sin \omega t$ and arrived at $I = I_o \sin\left(\omega t - \frac{\pi}{2}\right)$, the wave and phase diagram are;



Wave-diagram



Phase-diagram

AC circuit containing capacitor only.

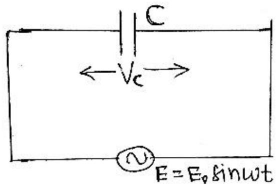


Fig: AC circuit containing capacitor only.

Let us consider a capacitor of capacitance C is connected with an AC source of emf $E = E_o \sin \omega t$. Let V_c be the potential across the capacitor. From Kirchhoff's second law (KVL),

$$E = V_c$$

Or, $\quad V_c = E_o \sin \omega t$

Let us consider a small amount of charge (dq) flows in small time (dt). Now, the current is

$$I = \frac{dq}{dt}$$

Or, $\quad I = \frac{d(V_cC)}{dt} \quad$ since, q = VC

Or, $\quad I = \frac{d}{dt} (C E_o \sin \omega t)$

Or, $\quad I = E_o C \frac{d}{dt} (\sin \omega t)$

Or, $\quad I = E_o C (\omega \cos \omega t)$

Or, $\quad I = E_o \omega C \cos \omega t$

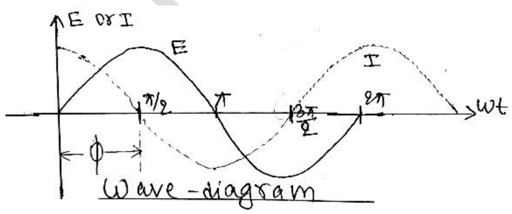
Or, $\quad I = \frac{E_o}{1/\omega C} \cos \omega t$

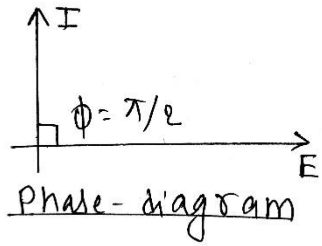
Or, $\quad I = \frac{E_o}{X_L} \cos \omega t$ (Where $X_c = \frac{1}{\omega c}$ is capacitive reactance

i.e. resistance offered by capacitor in a.c. circuit)

Or, $\quad I = I_o \cos \omega t$

Or, $\quad I = I_o \sin \left(\omega t + \frac{\pi}{2}\right)$





Since, we have started from $E = E_o \sin \omega t$ and arrived at $I = I_o \sin \left(\omega t + \frac{\pi}{2}\right)$, the wave and phase diagram are;

AC circuit containing resistor and inductor in series.

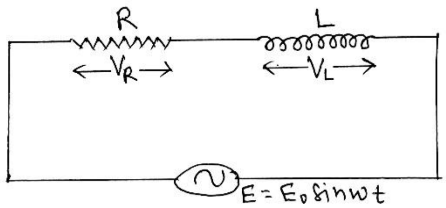
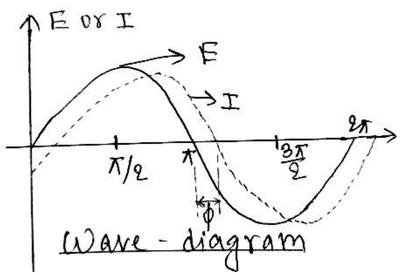
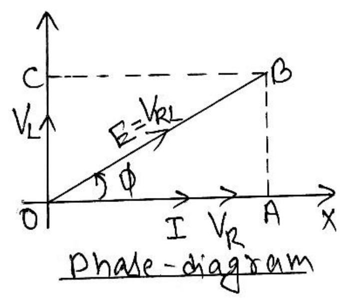


Fig: AC circuit containing resistor and inductor in series.

Let us consider a resistor of resistance R and an inductor of inductance L are connected in series with an AC source of emf $E = E_o \sin \omega t$. Let V_P and V_L be the potential across resistor and inductor respectively.





In AC circuit containing resistor only the current I and voltage V_R will in the same phase. The voltage V_R is reactant by OA in the figure.

In an AC circuit containing an inductor only, the voltage leads the current by phase angle $\frac{\pi}{2}$. The voltage V_L is represented by OC in the figure.

From ΔOAB,

$(OB)^2 = (OA)^2 + (AB)^2$

Or, $(OB)^2 = (OA)^2 + (OC)^2$

Or, $V_{RL}^2 = V_R^2 + V_L^2$

Or, $V_{RL}^2 = I^2R^2 + I^2X_L^2$ (X_L = Inductive reactance)

Or, $V_{RL}^2 = I^2 (R^2 + X_L^2)$

Or, $\frac{V_{RL}^2}{I^2} = R^2 + X_L^2$

Or, $Z^2 = R^2 + X_L^2$ (where $Z = \frac{V_{RL}}{I}$ is known as impedance i.e. total resistance offered by a.c. circuit.)

Or, $Z = \sqrt{R^2 + X_L^2}$

Calculation of phase angle (ϕ):

In figure, $\angle AOB = \phi$ is the phase angle.

From ΔOAB

$$\text{Tan}\,\phi = \frac{AB}{OA} = \frac{OC}{OA}$$

Or, $$\text{Tan}\,\phi = \frac{V_L}{V_R}$$

Or, $$\text{Tan}\,\phi = \frac{I\,X_L}{IR}$$

Or, $$\text{Tan}\,\phi = \frac{X_L}{R}$$

$\therefore$ $$\left[\phi = \tan^{-1}\left(\frac{X_L}{R}\right)\right]$$

The above result shows that voltage leads the current by phase angle ϕ, i.e. current lags behind the voltage by phase angle ϕ.

AC circuit containing resistor and capacitor in series.

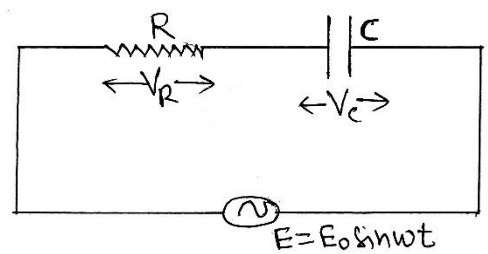
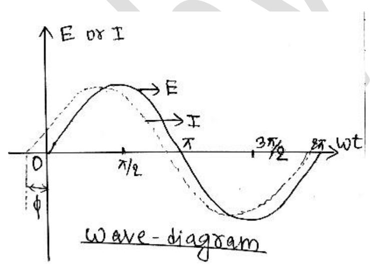
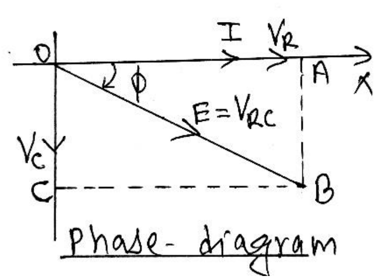


Fig: AC circuit containing resistor and capacitor in series

Let us consider a capacitor of capacitance C and a resistor of resistance R are connected in series with an AC source of emf

$E = E_0 \sin \omega t$. Let $V_R = IR$ and $V_C = IX_L$ be the potential across resistor and capacitor, respectively.





When an AC circuit is connected with a resistor only, the current I and voltage V_R will be in the same phase. The voltage V_R represented by OA in the figure.

When an AC circuit is connected with a capacitor only, the voltage lags behind the current by phase angle $\frac{\pi}{2}$. The voltage V_C is represented by OC.

Let us complete a parallelogram OABC. Now, the diagonal OB represents effective voltage E (i.e. V_{RC}).

From ΔOAB,

$$(OB)^2 = (OA)^2 + (AB)^2$$

or, $$(OB)^2 = (OA)^2 + (OC)^2$$

or, $$E^2 = V_R^2 + V_C^2$$

or, $$E^2 = (IR)^2 + (IX_C)^2$$

or, $$\frac{E^2}{I^2} = R^2 + X_C^2$$

or, $$Z^2 = R^2 + X_C^2 \quad \left(\text{where } Z = \frac{E}{I} \text{ is impedance total resistance offered by a.c. circuit.}\right)$$

$$\therefore \quad Z = \sqrt{R^2 + X_C^2}$$

Calculation of phase angle:

From ΔOAB,

$$\text{Tan } \phi = \frac{AB}{OA} = \frac{OC}{OA}$$

or, $$\text{Tan } \phi = \frac{V_C}{V_R}$$

or, $$\text{Tan } \phi = \frac{I X_C}{IR}$$

or, $$\text{Tan } \phi = \frac{X_C}{R}$$

$$\therefore \quad \left[\phi = \tan^{-1} \frac{X_C}{R}\right]$$

The above result shows that in an AC circuit containing capacitor and resistor in series, voltage lags behind the current by phase angle ϕ.

AC circuit containing inductor, capacitor and resistor in series. (LCR in series).

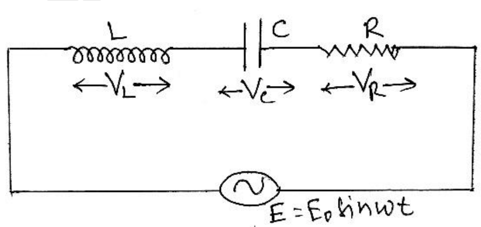
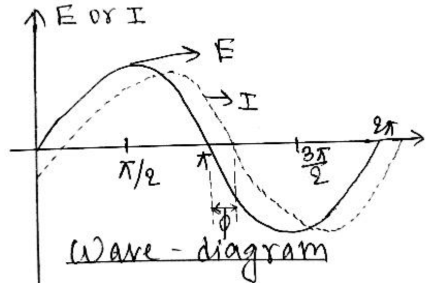
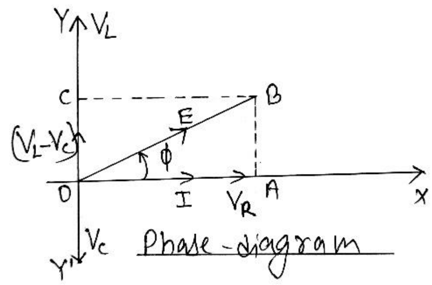


Fig: AC circuit containing inductor, capacitor and resistor in series

Let us consider an inductor of inductance L, a capacitor of capacitance C and a resistor of resistance R are connected in series with an AC source of emf, $E = E_o \sin \omega t$. Let V_L, V_C and V_R be the potential across inductor, capacitor and resistor, respectively.





In AC circuit containing resistor only, the current and voltage will be in the same phase. The voltage V_R is represented by OA in the figure.

In AC circuit containing inductor only, the voltage leads the current by phase angle $\frac{\pi}{2}$, the voltage V_L is represented by OY' in the figure and in AC circuit containing capacitor only, the voltage lags behind the current by phase angle $\frac{\pi}{2}$, the voltage V_C is represented by OY in the figure, The voltage V_L and V_C are in opposite direction. If ($V_L > V_C$) then the resultant voltage of V_L and V_C i.e., ($V_L - V_C$) is represented by OC.

Let us complete a parallelogram OABC. Now the diagonal OB represents the effective voltage E of the LCR series circuit.

From ΔOBA,

$$(OB)^2 = (OA)^2 + (AB)^2$$

or, $$(OB)^2 = (OA)^2 + (OC)^2$$

or, $$E^2 = V_R^2 + (V_L - V_C)^2$$

or, $$E^2 = I^2R^2 + (I X_L - I X_C)^2$$

or, $$\frac{E^2}{I^2} = R^2 + (X_L - X_C)^2$$

or, $$Z^2 = R^2 + (X_L - X_C)^2$$ ($\because$ $Z = \frac{E}{I}$ is impedance of AC circuit i.e. total resistance offered by AC circuit)

or, $$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

Calculation of phase angle:

From ΔOAB,

$$\text{Tan } \phi = \frac{AB}{OA} = \frac{OC}{OA}$$

or, $$\text{Tan } \phi = \frac{V_L - V_C}{V_R}$$

or, $$\text{Tan } \phi = \frac{I X_L - IX_C}{IR}$$

or, $$\text{Tan } \phi = \frac{X_L - X_C}{R}$$

$\therefore$ $$\left[\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)\right]$$

Special cases:

I. If $X_L > X_C$

In this case, the value of phase angle ϕ is positive i.e. voltage leads the current by phase angle ϕ.

II. If $X_L = X_C$

In this case, the value of phase angle ϕ is O. i.e. voltage and current will be in the same phase.

III. If $X_C > X_L$

In this case, the phase angle ϕ is negative i.e. voltage lags behind the current by phase angle ϕ.

Power consumed by AC circuit (LCR in series):

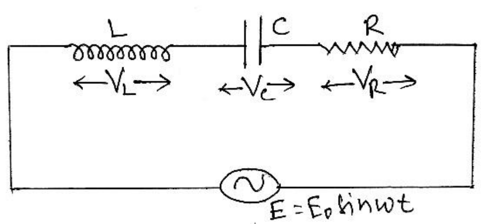


Fig: AC circuit containing inductor, capacitor and resistor in series

Let an inductor of inductance L, a capacitor of capacitance C and a resistor of resistance R are connected in series with an AC source. Let I and E be the rms value of current and emf, respectively. Let us consider the case in which current lags behind the voltage by phase angle ϕ. The instantaneous power developed is

$$P_{ins} = I.E.$$

or, $P_{ins} = I_o \sin(\omega t - \phi) \,.\, E_o \sin \omega t$

or, $P_{ins} = I_o E_o (\sin \omega t. \cos \phi - \cos \omega t \sin \phi) \sin \omega t$

or, $P_{ins} = I_o E_o (\sin^2 \omega t. \cos\phi - \sin \omega t \cos \omega t \sin \phi)$

Let dw be the small amount of work done in small time dt

i.e. $dw = P_{ins} \,.\, dt$ ………………… (i)

Now, the total amount of work done in a complete cycle can be obtained by integrating equation (i) from 0 to T.

or, $\int_O^T dw = \int_O^T P_{ins} \,.\, dt$

or, $W = I.\,E. \int_O^T (\sin^2 \omega t \cos \phi - \sin \omega t \cos \omega t \,.\, \sin\phi)\, dt$

or, $W = I.\,E. \left[\cos \phi \int_O^T \left(\frac{1 - \cos 2\omega t}{2}\right) dt - \frac{\sin \phi}{2} \int_O^T \sin 2\omega t \,.\, dt\right]$

or, $W = \frac{I_o\, E_o}{2} \left[\cos \phi \left\{\int_O^T dt - \int_O^T \cos 2\omega t\, dt\right\} - \sin \phi \,.\, 0\right]$ $[\because \int_O^T \text{Sin } 2\omega t\, dt = 0]$

or, $W = \frac{I_o\, E_o}{2} [\cos \phi \{T - O\}]$ $[\because \int_O^T \cos 2\omega t\, dt = 0]$

or, $W = \frac{I_o\, E_o}{2} \,.\, \cos \phi \,.\, T$

or, $W = \frac{I_o}{\sqrt{2}} \cdot \frac{E_o}{\sqrt{2}} \cos\phi\, T$

or, $W = I_{rms} \cdot E_{rms} \cos\phi\, T$

Now, the average power developed in complete cycle is

$$P_{avg} = \frac{W}{T}$$

$\therefore$ $P_{avg} = I_{rms} E_{rms} \cdot \cos\phi$

Where, $\cos\phi = \frac{R}{Z}$ is known as power factor.

Special cases

I. Resistor only

We have, $P_{avg.} = I_{rms} E_{rms} \cos\phi$

or, $P_{avg.} = I_{rms} E_{rms} \cos O \;\; (\phi = 0°)$

or, $P_{avg.} = I_{rms} E_{rms}$

II. Inductor only

We have, $P_{avg.} = I_{rms} E_{rms} \cos\phi$

or, $P_{avg.} = I_{rms} E_{rms} \cos\frac{-\pi}{2} \left(\because \phi = \frac{-\pi}{2}\right)$

$\therefore$ $P_{avg.} = 0$

III. Capacitor only

We have, $P_{avg.} = I_{rms} \cdot E_{rms} \cos\phi$

or, $P_{avg.} = I_{rms} \cdot E_{rms} \cos\left(\frac{\pi}{2}\right) \quad \left[\because \phi = \frac{\pi}{2}\right]$

or, $P_{avg.} = 0$

IV. L and R in series

We have, $P_{avg.} = I_{rms} E_{rms} \cos\phi$

$$\cos\phi = \frac{R}{Z}$$

$$Z = \sqrt{R^2 + X_L^2}$$

V. R and C in series

We have, $P_{avg.} = I_{rms} \; E_{rms} \cos\phi$

$$\cos\phi = \frac{R}{Z}$$

$$Z = \sqrt{R^2 + X_C^2}$$

Electrical resonance:

In an AC circuit containing L, C and R in series if $X_L = X_C$, the value of impedance becomes minimum and current becomes maximum. This condition is known as electrical resonance. In electrical resonance, current and voltage will be in same the phase.

For electrical resonance,

$$X_L = X_C$$

or, $$\omega L = \frac{1}{\omega C}$$

or, $$2\pi f_o L = \frac{1}{2\pi f_o C}$$

or, $$f_o^2 = \frac{1}{4\pi^2 LC}$$

or, $$f_o = \frac{1}{2\pi}\sqrt{\frac{1}{LC}}$$

Where f_o is known as resonance frequency.

Quality factor:

Quality factor is defined as the ratio of potential difference across inductor or capacitor to the potential difference across resistor in resonance condition.

$$\text{Q. factor} = \frac{\text{P.d. across inductor}}{\text{P.d. across resistor}}$$

$$= \frac{V_L}{V_R}$$

$$= \frac{I X_L}{I R}$$

$$= \frac{\omega L}{R}$$

$$= \frac{2\pi f o L}{R}$$

$$= 2\pi \cdot \frac{1}{2\pi}\sqrt{\frac{1}{LC}} \cdot \frac{L}{R}$$

$$\text{Q. factor} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

OR

$$\text{Q. factor} = \frac{\text{P.d. across capacitor}}{\text{P.d. across resistor}}$$

$$= \frac{V_C}{V_R}$$

$$= \frac{I X_C}{I R}$$

$$= \frac{1/\omega C}{R}$$

$$= \frac{1}{2\pi fo\ CR}$$

$$= \frac{1}{2\pi . \frac{1}{2\pi} \sqrt{\frac{1}{LC}} . CR}$$

$$\text{Q. factor} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

Choke coil:

A coil having a large number of turns and negligible internal resistance is known as a choke coil. It is a pure inductor. Average power consumed in AC circuit containing inductor only is

$$P_{avg.} = I_{rms}\ E_{rms} \cos\phi$$

$\because$ For inductor only $\phi = \frac{\pi}{2}$

$$P_{Avg.} = I_{rms}\ E_{rms} \cos\frac{\pi}{2}$$

$\therefore$ $P_{Avg.} = O$

Wattless current:

In an AC circuit containing an inductor only or capacitor only, the value of phase angle is $\frac{\pi}{2}$ and average power consumed is O. Current in this case is known as wattles current.

Wattful current:

In an AC circuit containing resistor only, the average power consumed is $I_{rms}\ E_{rms}$. The current in this case is known as watt full current.

Admittance:

Admittance is defined as the reciprocal of impedance. It is denoted by Y.

Mathematically,

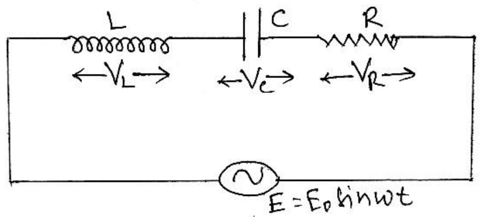
$$\text{Admittance, } Y = \frac{1}{\text{Impedance (Z)}}$$

$$Y = \frac{1}{\sqrt{R^2 + (X_L - X_C)^2}}$$

Its unit is Ω^{-1}.

Alternating Current

Formula for Numerical Problems:



1. The instantaneous value of alternating current is

$$I = I_o \sin \omega t.$$

Where I_o is maximum (peak) value of current.

And the instantaneous value of alternating voltage is

$$E = E_o \sin \omega t.$$

Where E_o is maximum (peak) value of voltage.

2. For certain phase angle (ϕ)

$E = E_o \sin \omega t$ and $I = I_o \sin (\omega t \pm \phi)$ where ϕ is phase angle between emf and current.

3. $I_{rms} = \dfrac{I_o}{\sqrt{2}}$ & $E_{rms} = \dfrac{E_o}{\sqrt{2}}$

4. Phase angle (ϕ) between current and voltage in AC circuit containing

R	**L**	**C**	**R & L**	**R & C**	**L,C,R**
$\phi = 0$	$\phi = -\dfrac{\pi}{2}$	$\phi = \dfrac{\pi}{2}$	$\tan \phi = \dfrac{X_L}{R}$	$\tan \phi = \dfrac{X_C}{R}$	$\tan \phi = \dfrac{X_L - X_C}{R}$
i.e. I & E are in same phase	i.e. I lags behind the E by $\dfrac{\pi}{2}$ (i.e. by 90°)	i.e. I leads the E by $\dfrac{\pi}{2}$ (i.e. by 90°)	i.e. I lags behind the E by $\dfrac{\pi}{2}$ (i.e. by 90°)	i.e. I leads the E by phase angle ϕ.	For $X_L > X_C$ i.e. I lags behind the E by phase angle ϕ.

5. Inductive reactance (i.e. resistance offered by inductor),

$$X_L = \omega L = 2\pi f L$$

Capacitive reactance (i.e. resistance offered by inductor),

$$X_c = \frac{1}{\omega c} = \frac{1}{2\pi f C}$$

6. Impedance of AC circuit i.e. total resistance offered by AC circuit

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

7. Voltage drop across R, $V_R = IR$

Voltage drop across L, $V_L = I\, X_L$

Voltage drop across C, $V_C = I\, X_c$

8. Circuit Formula $E = IZ$ (in AC) [$E=V=I\, R_{eq}$, for $r = 0$ (in D.C)]

9. Electrical Resonance

At resonance, I is max^m and hence Z is min^m as $E= IZ$

We have, Impedance, $Z = \sqrt{R^2 + (X_L - X_C)^2}$

i.e. at resonance, $X_L = X_c$

Resonating frequency, $f_o = \frac{1}{2\pi}\sqrt{\frac{1}{LC}}$

10. Quality factor $= \frac{V_L \text{ or } V_C}{V_R}$

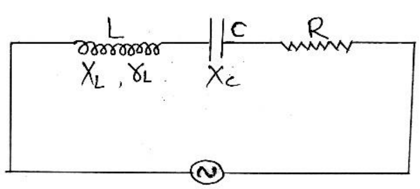
Q. factor $= \frac{1}{R}\sqrt{\frac{L}{C}}$

11. Average power consumed in AC circuit

$P_{avg} = I_{rms}\, E_{rms} \cdot \cos\phi$

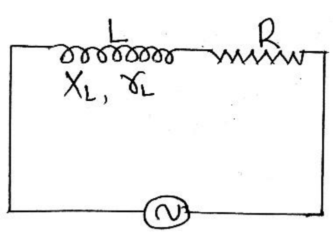
12. Power factor, $\cos\phi = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + (X_L - X_C)^2}}$

13. A.C. Circuit containing inductor with some internal resistance (r_L)



(i) Impedance of AC circuit, $Z = \sqrt{(R + r_L)^2 + (X_L - X_C)^2}$

(ii) Impedance of AC circuit containing L & R, $Z = \sqrt{(R+r_L)^2 + X_L^2}$



(iii) Total resistance offered by inductor, $Z_L = \sqrt{r_L^2 + X_L^2}$

(iv) Voltage drop across inductor, $V_L = I\, Z_L$

(v) Phase angle (ϕ), $\tan\phi = \frac{Z_L}{R}$